Introduction to Game Theory

Instructor: Vassili Kolokoltsov

1. Abstract

Game theory is the mathematical discipline aimed to model various interactions of living organisms in quantitative terms. Game theory, as the universal method for the analysis of social interactions, has wide applications in economics, in the theory of control and management, financial mathematics, evolutionary biology, sociology, psychology and politics, in modelling different social processes, in particular, the processes of democratic elections, processes of fair distributions of resource, processes of arms control, etc. The course is designed for all wishing to get acquainted with main ideas and methods of game theory.

Game theory is a mathematical discipline. Therefore, for the fully-fledged understanding one has to have at least basic knowledge of mathematical analysis, linear algebra, differential equations and probability theory. However, many ideas of game theory can be explained without the use of serious mathematics. In order to be more accessible to a wider auditorium, the first part of the course is designed specifically to the explanation of the basic ideas without any advanced mathematics. Here we also spend some time on historical aspects related to lives of the founders of the theory. The requirements to the mathematical education of the auditorium increase to the second part of the course.

The course is capacious and covers a wide circle of problems and notions. They include the Nash equilibrium, auctions, Braess paradox, selfish routing, method of backward induction, models of voting and fair distributions, evolutionary games, evolutionary stable strategies, dynamic programming, Hamilton-Jacobi-Bellman equation, infinite time games and computer tournaments. Also covered are the pricing of financial instruments (options and credit derivatives), Black-Scholes theory and game options, games with a large number of players in statistical limit, mean field games, models of cooperation and coalition building. Examples include the games of the arms race, exploitation of common resources, social dilemmas (battle of the sexes, sex ratio game, sacrifice game, models of inspection and corruption, modelling antiterrorist measures, as well as biological and genetic information transmission.

2. Course Contents

Part 1. Ideas, methods, applications (6 hours lectures, 6 hours seminars).

Chapter 1. Around Prisoner's dilemma: static games of two players.
Chapter 2. Auctions and networks: static games of several players.
Chapter 3. Backward induction and repeated games.
Chapter 4. Aggregation of preferences: elections, social agreement, fair distribution.

Part 2. Foundations of non-cooperative games. Basic mathematical methods and application to evolutionary biology. Equilibria, dynamics and evolutionary stable strategies (8 hours lectures, 8 hours seminars).

Chapter 1. Nash equilibria for static games with a finite strategy space.
Chapter 2. Evolutionary stable strategies (ESS) and replicator dynamics (RD).
Chapter 3. Dynamic games and dynamic programming.
Chapter 4. Games with a continuous state space.

Part 3. Game-Theoretic approach to financial mathematics. Pricing derivative securities (10 hours lectures, 10 hours seminars).

Chapter 1. Preliminaries: geometric theory of risk-neutral measures.
Chapter 2. Game-theoretic origins of risk-neutral laws.
Chapter 3. Rainbow options in discrete times.
Chapter 4. Continuous time limit: generalized Black-Scholes equations.
Chapter 5. Pricing credit derivatives.
Chapter 6. Comparison with the standard probabilistic approach.
Chapter 7. Dynkin's games and game options.

Part 4. Games of many players in statistical limit. Mean field games (10 hours lectures, 10 hours seminars).

Chapter 1. Introduction: Main ideas and applications of the statistical limit.
Chapter 2. Dynamic law of large numbers (LLN): rigorous results.
Chapter 3. Mean-field type dynamic control with major players.
Chapter 4. Mean-field games (MFGs) for finite-state models.
Chapter 5. Simplest (3-state and 4-state) models of MFGs.

3. Assessment Elements and Grading System:

0.3 Activity on Seminars+ 0.7 * Exam

Student performance could be measured on a 10-point scale that translates to the 5-point grading scale as follows:

8-10	5	Excellent
6-7	4	Good
4-5	3	Satisfactory
0-3	0-2	Fail

4. Reading List

a. Required

- V. N. Kolokoltsov, O. A. Malafeyev. Understanding Game Theory. World Scientific 2010. Second Edition 2020.
- V. N. Kolokoltsov and O. A. Malafeyev (2019). Many Agent Games in Socio-economic Systems: Corruption, Inspection, Coalition Building, Network Growth, Security}. Springer Series in Operations Research and Financial Engineering, Springer Nature, 2019. http://doi.org/10.1007/978-3-030-12371-0

b. Optional

- P. Bernhard, J. Engwerda, B. Roorda, J.M. Schumacher, V. N. Kolokoltsov, P. Saint-Pierre and J.-P. Aubin. The Interval Market Model in Mathematical Finance: Game-Theoretic Methods. Birkh\"auser, 2012.
- Hans Föllmer, Alexander Schied. Stochastic finance: an introduction in discrete time (2016). Fourth revised and extend edition.

- Yuri Kifer (2013). Dynkin’s games and Israeli options. ISRN Probability and Statistics
- Volume 2013, Article ID 856458, 17 pages, http://dx.doi.org/10.1155/2013/856458
- R. Carmona and F. Delarue (2018). Probabilistic Theory of Mean Field Games with Applications, v. I, II. Probability Theory and Stochastic Modelling v. 83, 84. Springer.