

August 23d - August 27th

Dmitry Kramkov

Topic: Arbitrage-free pricing, optimal investment and equilibrium

Andreas Hamel

Topic: Set-valued methods in finance and economics: what to do when the preference/order relation is no longer complete

Çağın Ararat

Topic: Set-valued analysis with applications in Economics and Finance

Mathieu Rosenbaum

Topic: Rough volatility, theory and practice

Christa Cuchiero

Topic: From neural SDEs and signature based models to affine and polynomial processes and back

Vassili Kolokoltsov

Topic: Games of many players: modelling of socio-economic processes

Oleg Kudryavtsev

Topic: Option pricing under Levy models



Abstracts



Andreas Hamel Free University of Bozen-Bolzano

Set-valued methods in finance and economics: what to do when the preference/order relation is no longer complete.

Prerequisites: basics on order relations, lattices and ordered vector spaces including convex sets and cones (not in class, material is provided)

Introduction : (1) complete lattices of sets based on set relations, "upper" and "lower" sets, examples (math examples as well as buyer/seller prices, quantile sets etc.), (2) the conlinear space structure as an appropriate generalization of linear spaces, residuated conlinear spaces, (3) why a "pure" vector/multi-criteria approach doesn't work.

First topic: functions with values in complete lattices of sets with special emphasis on T-translative functions, properties ((quasi)convexity, sublinearity, monotonicity, T-translativity, semicontinuity), representation via families of scalar functions, examples (sub/superhedging portfolios in markets with transaction costs, set-valued quantiles and lower/upper expectations for multivariate r.v.s, projections and aggregation functions, set-valued risk measures).

Second topic: set-valued duality theory and applications to risk measures and beyond (with the Kabanov superhedging theorem as a prominent example), in particular the case of convex T-translative functions in finance.

Third topic: set-valued treatment of multi-utility maximization problems; from multi-utility representations to set-valued utility maximization via closure operators, solution concepts and existence, multidimensional (optimized) certainty equivalents.

Optional: an outlook to the theory of games with multi-dimensional payoffs based on the complete lattice approach, new equilibrium concepts and ramifications.



Dmitry Kramkov

Carnegie Mellon University

The goal of the lectures is to provide an introduction to the three basic topics in asset pricing theory which are arbitrage, single-agent optimality, and equilibrium. The outline of the course is the following:

Part 1: Arbitrage-free pricing.

1. Absence of arbitrage and 1st fundamental theorem. 2. Risk-neutral valuation. 3. Completeness and 2nd fundamental theorem.

Part 2: Optimal investment. 1. Preferences and utility functions. 2. Merton problem: PDE approach. 3. Duality and martingale measures.

Part 3: Equilibrium. 1. Pareto optimality. 2. (Static) Arrow-Debreu equilibrium. 3. (Dynamic) Radner equilibrium.



Çağın Ararat

Bilkent University, Ankara, Turkey

- Tentative Outline
- Lecture 1: Set-valued risk measures: static case (2 hours)
- Univariate static risk measures
- Acceptance sets, primal representations
- Coherent, convex, quasiconvex cases
- Clearing models
- Systemic risk measures

Lecture 2: Set-valued risk measures: dynamic case (2 hours)

- Univariate dynamic risk measures
- Connection to backward stochastic differential equations
- Multiportfolio time-consistency
- Set-valued dynamic programming principle
- Backward stochastic difference inclusions (BSDI)
- Extension to continuous time: open problems

Lecture 3: Side benefits of set-valued analysis for time-inconsistency (2 hours)

- Mean-risk problem
- Mean-variance problem

Lecture 4: Set-valued stochastic analysis (4 hours)

- Convex compact sets, Hukuhara difference
- Measurability, decomposability
- Set-valued expectation, conditional expectation
- Set-valued stochastic processes
- Aumann-Lebesgue and Aumann-Itô integrals
- Lack of additivity, unboundedness
- Set-valued martingales
- Martingale representation theorem
- Set-valued backward stochastic differential equations



Mathieu Rosenbaum Ecole Polytechnique, Paris



Abstracts



Christa Cuchiero

University of Vienna From neural SDEs and signature based models to affine and polynomial processes and back

Modern universal classes of dynamic processes, based on neural networks or signature methods, have recently entered the field of stochastic modeling, in particular in Mathematical Finance. This has opened the door to more data-driven and thus more robust model selection mechanisms, while first principles like no arbitrage still apply. The underlying model classes are often so-called neural stochastic differential equations (SDEs) or signature based models, i.e. models whose characteristics are either neural networks or linear functions of the process' signature. We present methods how to learn these characteristics from available option price and time series data.

From a more theoretical point of view, we show how these models can be embedded in the framework of affine and polynomial processes, which have been — due to their tractability — the dominating process class prior to the new era of highly over-parametrized dynamic models. Indeed, we prove that generic classes of jump diffusion models can be viewed as infinite dimensional affine processes, which in this setup coincide with polynomial processes. A key ingredient to establish this result is again the signature process. This then allows to get power series expansions for expected values of analytic functions of the process' marginals, which also apply to neural or signature SDEs. In particular, expected signature can be computed via polynomial technology.



Oleg Kudryavtsev

The Wiener-Hopf factorization method for pricing barrier options: a simple numerical realization.

The paper suggests a new approach to pricing barrier options under pure non-Gaussian Levy processes with jumps of finite variation. The key idea behind the method is to represent the process under consideration as a difference between subordinators (increasing Levy processes). Such splitting rule applied to the process at exponentially distributed randomized time points gives us the possibility to find the option price by analytically solving simple Wiener-Hopf equations.



Vassili N. Kolokoltsov University of Warwick Games of many players: modelling of socio-economic processes

Game theory is the mathematical discipline aimed to model various interactions of living organisms in quantitative terms.

The general picture we shall deal in these lectures is characterized by a set of big players, also referred to as principals or major agents, acting on the background of large pools of small players. The impact of the behaviour of each small player in a group on the overall evolution is decreasing with the increase of the size of the group. The examples of the real world problems involved include government representatives (often referred to in the literature as benevolent dictators) chasing corrupted bureaucrats, inspectors chasing tax-paying avoidance, police acting against terrorist groups or models describing the attacks of computer or biological viruses. This includes the problem of optimal allocation of the budget or efforts of the big player to different strategies affecting small players, for instance, the allocation of funds (corrected in real time) for the financial support of various business or research projects. Other class of examples concerns appropriate (or better optimal) management of complex stochastic systems consisting of large number of interacting components (agents, mechanisms, vehicles, subsidiaries, species, police units, robot swarms, etc), which may have competitive or common interests. Such management can also deal with the processes of merging and splitting of functional units (say, firms or banks) or the coalition building of agents.

In Lecture I we shall discuss the general principles underlying the theory, namely the dynamic law of large numbers that allows one to reduce a complex stochastic process of interaction to a deterministic dynamic, in the limit of infinite number of players.

In Lecture II we give some basic mathematical result and investigate in more detail a concrete example, a model of the processes of corruption and inspection, and discus the clear practical conclusions that can be derived from this analysis.

In Lecture III we give an introduction to the so-called mean-field games, one of the central new branch (actually about only 15 year old) of the gamer theory.

In the lectures, we will mostly develop the ideas from the recent monograph V. N. Kolokoltsov and O. A. Malafeyev: Many Agent Games in Socio-economic Systems: Corruption, Inspection, Coalition Building, Network Growth, Security. Springer Series in Operations Research and Financial Engineering, Springer Nature, 2019.

